Revision problems

1. The cubic polynomial $2x^3 + ax^2 + b$ is denoted by $f(x)$. It is given that $(x+1)$ is a factor of $f(x)$, and that when $f(x)$ is divided by $(x+2)$ the remainder is -5. Find the values of a and b.

2. The polynomial $x^4 - 6x^2 + x + a$ is denoted by $f(x)$.
   (a) It is given that $(x+1)$ is a factor of $f(x)$. Find the value of a.
   (b) When a has this value, verify that $(x-2)$ is also a factor of $f(x)$ and hence factorize $f(x)$ completely.

3. Solve the equation $|x-1| + |2x+1| = x+4$.

4. Solve the following inequalities:
   (a) $|x-4| > x+1$,
   (b) $|2x-1| < |3x|$.

5. Expand $(1-3x)^{-\frac{1}{3}}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients.

6. Express $f(x) = \dfrac{4x}{(3x+1)(x+1)^2}$ in terms of partial fractions.

7. Express $f(x) = \dfrac{1}{(3x+1)(2x^2+x+1)}$ in terms of partial fractions. Hence expand $f(x)$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients.

8. Solve the following equations:
   (a) $5^{x-2} = 5^x - 2$; giving your answer correct to 3 significant figures,
   (b) $\ln(e^x + 1) = 2\ln(e^x - 1)$; giving your answer in an exact form.

9. The variable x and y satisfy the equation $y^3 = Ae^{2x}$, where A is a constant. The graph of $u = \ln y$ against x is a straight line.
   (a) Find the gradient of this line.
   (b) In the u-x diagram, given that this line intersects the u-axis at the point where $u = 0.5$, find the value of A, correct to 2 decimal places.

10. (a) Prove the identity $\csc 2x - \cot 2x = \tan x$.
    (b) Use this result to find the exact value of $\tan 15°$, without using a calculator.

11. Express $3\sin x - 4\cos x$ in the form $R\sin(x-\alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Hence
    (a) solve the equation $3\sin x - 4\cos x = 2$, giving all solutions between 0 and 2π;
    (b) find the greatest and least values, as x varies, of the expression $\dfrac{1}{3\sin x - 4\cos x + 10}$.

12. Denote $\theta = \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$.
    (a) Use the addition formula of tangent twice to find $\tan\theta$.
    (b) Thus find the exact value of θ.